

CLAMP-ON, SONAR-BASED VOLUMETRIC FLOW RATE AND GAS VOLUME FRACTION MEASUREMENT FOR INDUSTRIAL APPLICATIONS

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1 ABSTRACT

A clamp-on, sonar-based flow measurement technology for simultaneously measuring volumetric flow rate and process fluid sound speed is described. The technology utilizes sonar array processing techniques to perform two independent measurements to characterize the process fluid, each well-suited for single and multiphase flows. Firstly, the meter provides mixture volumetric flow by tracking the speed at which naturally occurring flow-generated pressure fields convect past an array of strain-based sensors clamped-on to existing process pipe. Secondly, the meter provides compositional information by measuring the speed at which naturally occurring sound waves propagate through the process fluid using the same clamp-on sensors. Measuring the speed of sound of a process fluid utilizing this sonar technique is analogous to measuring process fluid density and has many compositional based applications. A primary example of this is using process sound speed to accurately measure the amount of entrained gases in liquid–continuous mixtures.

The ability to measure volumetric flow and process sound speed reliably and accurately in single and multiphase applications with a non-intrusive, clamp-on device significantly enhances the observability of the many industrial processes. Data is presented from sonar-based volumetric flow and sound speed-based compositional measurements for a wide range of industrial applications, encompassing single phase liquid applications to large diameter, abrasive multiphase applications, widely-recognized as difficult, long-standing flow measurement challenges.

2 INTRODUCTION

Volumetric flow is a critical measurement in process control and optimization for most industrial processes. The current industrial flow meter market is often classified into two technology-based categories: old technology and new technology. Old technology flow meters include flow measurement technologies that have been in use for more than 70 years. It includes turbine meters, orifice plates and variable area flow meters. The new technology flow meters include technologies which have emerged over the last 30~50 years. These new technologies typically offer advantages over the old technologies in performance, functionality, and reliability. The major types of new technology flow meters include ultrasonic meters, electromagnetic flow meters, vortex flow meters, and coriolis flow meters. Each type has evolved to serve various aspects of the diverse range of applications within the industrial flow meter landscape.

This paper describes sonar-based flow meter technology which utilizes sonar techniques¹ to listen to, and interpret, pressure fields generated by turbulent pipe flows. Sonar flow measurement technology represents a new class of industrial flow meters utilizing measurement principles distinct from existing technologies. Sonar flow meters were first introduced into the oil and gas industry in 1998 for use in downhole multiphase flow metering applications^{2 3}. Sonar flow measurement technology is currently being used in other industries such as pulp and paper, mining and minerals, hydro transport, chemicals and power generation.

Process aeration is often an unwanted but unavoidable phenomenon that negatively impacts product quality or the ability to accurately determine the rate of flow. For instance, entrained air in the thin stock flow to the headbox of a paper machine may lead to pin holes in the

produced paper or worse still to a web break resulting in production loss. Entrained air in the paper and pulp industry is such a pervasive problem⁴ that both mechanical and chemical means are used to remove air from the process. Likewise in production of crude oil, gas carry under in the liquid outlet of a twophase separator impairs the measurement of the rate of net oil flow as the watercut meter is affected by the presence of free gas. It is well known that the resulting errors in the net oil rate can easily be 100% or more solely because of a small error in the watercut.

Just as often, however, process aeration is an integral part of the production process and the air or gas content must be kept within certain limits. By way of example, the injection of a small amount of air in the hydrotransport pipe lines in oil sand processing initiates the separation of oil from sand. Likewise in the separation of rock from minerals in flotation cells, gas is sparged in at the bottom of the cell and the gas holdup in the cells must be controlled to optimize the gravity separation of rock from mineral.

The sonar techniques that enable the interpretation of the naturally occurring pressure fields inside a pipe are also well suited to determine the velocity of propagation of acoustic pressure fields. The velocity of sound thus determined is a very sensitive indicator of process aeration and it offers a superior alternative to today's mechanical, sample taking analyzers.

The methodology involves characterizing the speed at which coherent pressure waves, either vortical structures or acoustic pressure fields, convect past an axial array of sensors using beam-forming techniques developed over several decades for underwater acoustic applications. Since coherent vortical structures are an inherent feature of turbulent boundary layers, no internal geometry is required to generate these structures. Likewise, in the majority of industrial processes the proximity of pumps, compressors, valves or sharp bends ensures that no source of sound is required. The sonar-based measurement is therefore entirely passive and it can be performed using an array of strain based sensors mounted on the outside of the pipe. With no process-wetted hardware, the sonar flow measurement technology is well suited for the corrosive and abrasive slurries commonly encountered in many practical applications.

3 FLOW RATE MEASUREMENT

The overwhelming majority of industrial process flows involve turbulent flow. Turbulent fluctuations within the process flow govern many of the flow properties of practical interest including the pressure drop, heat transfer and mixing. For these reasons, turbulent pipe flows have been extensively studied over the years with roots back to Osbourne Reynolds and Lord Rayleigh in the late nineteenth century⁵.

3.1 Turbulent pipe flow

For engineering applications, considering only the time averaged properties of turbulent flows is often sufficient for design purposes. For sonar flow metering technology, understanding the time averaged velocity profile in turbulent flow provides a means to interpret the relationship between speed at which coherent structures convect and the volumetrically averaged flow rate within a pipe. For turbulent flows, the time-averaged axial velocity varies with radial position, from zero the wall to a maximum at the centerline of the pipe. The flow near the wall is characterized by steep velocity gradients and transitions to relatively uniform core flow near the center of the pipe. Figure 1 shows a representative schematic of a velocity profile and coherent vortical structures present in fully developed turbulent pipe flow.



Figure 1: Coherent structures in turbulent pipe flows.

The vortical structures are superimposed over time averaged velocity profile within the pipe and contain temporally and spatially random fluctuations with magnitudes typically less than 10% percent of the mean flow velocity. The Reynolds number (Re), based on pipe diameter (D), characterizes many of the engineering properties of the flow. The Reynolds number is a non-dimensional ratio representing the relative importance of inertial forces to viscous forces within a flow: Pipe flows with Reynolds numbers exceeding a critical value, typically 2300, are turbulent. Those with Reynolds numbers below this value are laminar. The vast majority of flows in industrial processes is turbulent with Reynolds numbers far in excess of the critical value.

In addition to demarcating a boundary between laminar and turbulent flow regimes, the Reynolds number is a similarity parameter for pipe flows, i.e. flows in geometrically similar pipes, scaled with radius, with the same Reynolds number are dynamically similar⁶. Empirical studies have shown that velocity profiles in turbulent pipe flows are well represented by the 1 over nth power law:

$$\frac{u(r)}{U_0} = \left(1 - \frac{r}{R}\right)^{1/n} \quad (1),$$

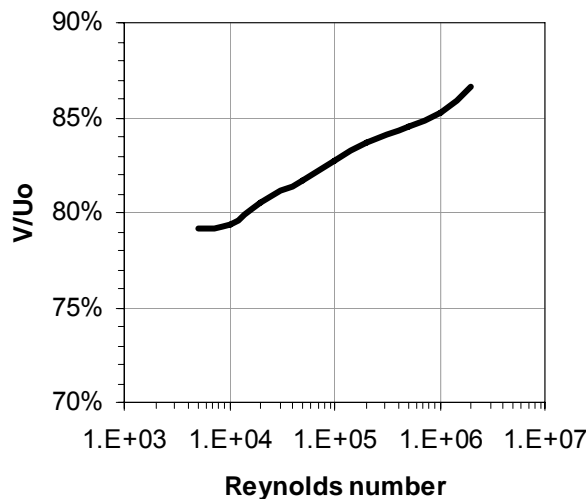


Figure 2: Mean velocity normalized by Maximum velocity vs. Reynolds number.

with n ranging from 6 to 10 as a weak function of Reynolds number. From a measurement perspective, the volumetrically averaged flow velocity V (mean velocity) or the volume rate of flow divided by the pipe's internal cross-sectional area is of interest. The relation between the ratio V/U₀ and the Reynolds number is shown in Figure 2, which is based upon the power law (1). Apparently the ratio of the mean flow velocity to the maximum velocity slightly increases from a little below 80% just above the critical Reynolds number for turbulent flow to greater than 85% at Reynolds numbers over one million. It must be stressed that, given the velocity profile (1), very little fluid is actually flowing at the mean flow velocity.

3.2 Coherent turbulent structures

Turbulent pipes flows are highly complex flows. Predicting the details of any turbulent flow is one of nature's great-unsolved problems. However, much is known regarding the statistical properties of the flow. For instance, turbulent pipe flows contain self-generating, coherent vortical structures often termed "turbulent eddies". The maximum length scale of these eddies is scales with the diameter of the pipe. These structures remain coherent for several pipe diameters downstream, eventually breaking down into progressively smaller eddies until the energy is dissipated by viscous effects. Experimental investigations have established that eddies generated within turbulent boundary layers convect at roughly 80% of maximum flow velocity. For pipe flows, this implies that turbulent eddies will convect at approximately the volumetrically averaged flow velocity within the pipe. The precise relationship between the convective velocity of turbulent eddies and the flow rate for each class of meters can be calibrated empirically as described below.

3.3 Characterizing the unsteady pressure field

The sonar flow metering methodology uses the convection velocity of coherent structure with turbulent pipe flows to determine the volumetric flow rate. The convection velocity of these eddies is determined by applying sonar arraying processing techniques to determine the speed at which eddies convect past an axial array of dynamic strain measurements

distributed along the pipe outer circumference. The sonar-based algorithms determine the speed of eddies by characterizing both the temporal and spatial frequency characteristics of the flow field. For a train of coherent eddies convecting past a fixed array of sensors, the temporal and spatial frequency content of pressure fluctuations are related through the following relationship:

$$\omega = kv \quad (2).$$

Here is the wave number, defined as $k=2\pi/\lambda$ (1/m), ω is the temporal frequency (rad/s) and v is the convective velocity of the unsteady pressure fluctuations. Thus, the shorter the wavelength is the larger k and the higher the temporal frequency.

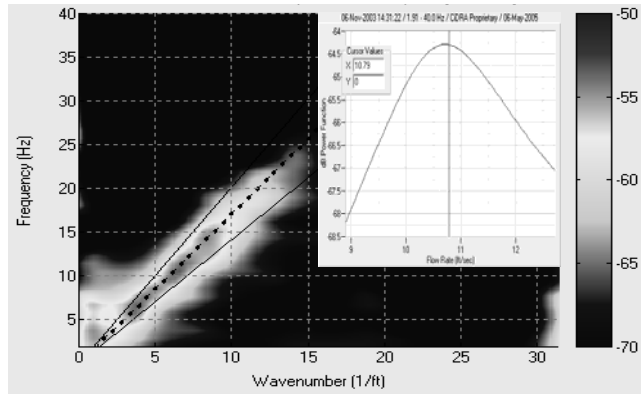


Figure 3: k-w plot showing convective ridge

ridge on a k-w plot indicates the convective velocity of the pressure field. This suggests that the convective velocity of turbulent eddies, and hence flow rate within a pipe, can be determined by constructing a k-w plot from the output of a phased array of sensors and identifying the slope of the convective ridge. Figure 3 shows an example of a k-w plot generated from a phased array of transducers listening to a 16 inch pipe flowing water at approximately 350 l/s. The power contours show a well-defined convective ridge. A parametric optimization method was used to determine the “best” line representing the slope of the ridge. For this case, a slope of 3.2 m/s was determined. The intermediate result of the optimization procedure is displayed in the insert, showing that optimized value is unique and constitutes of one well-defined maximum.

In sonar array processing, the spatial - temporal frequency content of time stationary sound fields are often displayed using “k-w plots”. k-w plots are essentially two dimensional power spectra in which the power of a pressure field is decomposed into bins corresponding to specific spatial wave numbers and temporal frequencies. On a k-w plot, the power associated with a pressure field convecting with the flow is distributed in regions which satisfy the dispersion relationship developed above. This region is termed the “convective” ridge and the slope of this

3.4 Sonar flow meter calibration

The k-w plot shown in Figure 3 illustrates the fundamental principle behind sonar based flow measurements, namely that axial arrays of transducers can be used in conjunction with sonar processing techniques to determine the speed at which naturally occurring turbulent eddies convect within a pipe. The next issue is to quantify the relationship between speed of the turbulent eddies and the volumetrically averaged flow rate within the pipe.

To quantitatively evaluate this relationship, a number of geometrically similar sonar flow meters with diameters between 4 and 16 inch were tested with water at a flow meter calibration facility for flows ranging from 1.5 to 1500 l/s. The convection velocity determined using the sonar-based techniques, normalized by the volumetrically averaged flow rate supplied by the calibration facility as a function of Reynolds number ranged between 99% and 102% of the volumetrically averaged flow rate over the entire range of the test. A low-order Reynolds number based calibration, was developed from this data for this class of meters. The calibration equation is developed in terms of the relative offset of the sonar based meter from the reference meter. The relative offset Δ is, in terms of the Reynolds number Re and a set of three coefficients C_1 , C_2 and C_3 , unique to each sonar meter size given by (3).

$$\Delta = C_1 + \frac{C_2}{Re^{C_3}} \quad (3).$$

Figure 4 shows the relative difference of the volumetric flow rate measured by the calibrated sonar meters and the reference flow plotted versus reference flow. Calibration data was recorded for the sonar flow meters with volumetrically averaged flow velocities ranging from 1-10 m/s. Using a single Reynolds number calibration (4) spanning the operating range of the three flow meters of different physical sizes, the sonar meter measured the volumetric flow rate

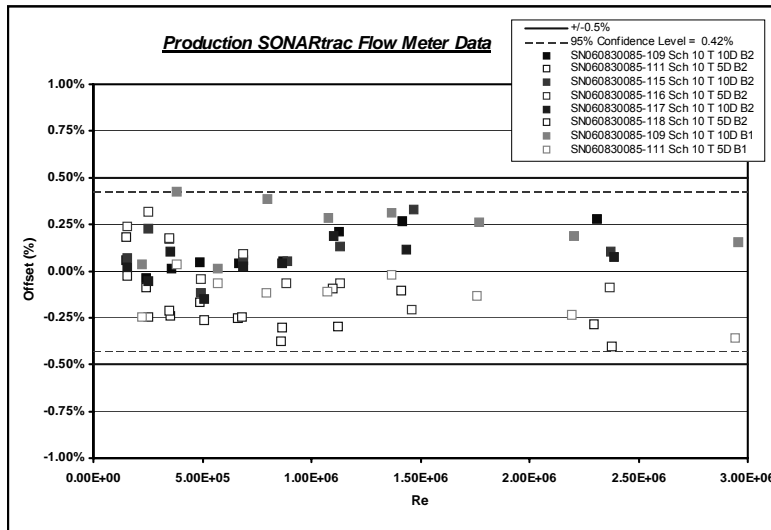


Figure 4: Calibration data.

concluded from the data presented that the 95% confidence level of the sonar based flow measurement equals 0.42%.

3.5 Industrial applications

Numerous industrial applications for the sonar based flow measurement have been developed in close co-operation with end users over a period of more than 2 years. The industrial applications are to be found where the clamp on, non intrusive, large bore capability of the sonar based principle results in undisputed advantages in terms of life cycle cost, reliability or the sheer impossibility to make any other measurement device work. Examples of such are:

- Hydro transport of oil sand⁸.
- Thick stock flow in paper and pulp⁹.
- Air conveyed transport of particulate material.
- Two phase flow of water and oil^{10 11}.
- Sewer water processing.
- Flow of coating pastes.
- Magnetite flow in minerals processing.

All of the above examples of industrial flows are characterized by their abrasive and/or corrosive nature. In many cases this necessitates the use of special pipe materials or the use of lined (Teflon, rubber, concrete) pipe. The phenomenon of pressure fluctuation induced circumferential pipe strain is universal and independent of the precise nature of the pipe material or lining. As a result a sonar based measurement can, in most cases, be applied without a process shut-down, without cutting the pipe and without elaborate operational procedures or safety measures.

In selected cases the operating principle of the sonar based meter removes a calibration difficulty inherent in the traditional measurement principle. For instance, in the flow rate measurement of magnetite suspensions by electromagnetic flow meters the ferro magnetic nature of the suspended particulates affects the meter's calibration. As a result the meter may drift with magnetite concentration. Figure 5 shows two frequency distributions of the relative difference between a sonar based and electromagnetic flow rate measurement for both high (1.70 kg/dm³) and low (1.65 kg/dm³) density magnetite suspensions in the feed flow to a

rate to within 0.5% accuracy. It is important to note that this flow metering approach has no fundamental size limitations and should be applicable to turbulent pipe flows of all diameters and Reynolds numbers. Furthermore, similarity laws suggest, and data from Figure 4 support, that the relationship between convection velocity and flow rate from geometrically similar meters of any size is governed by same Reynolds number based calibration. It can be

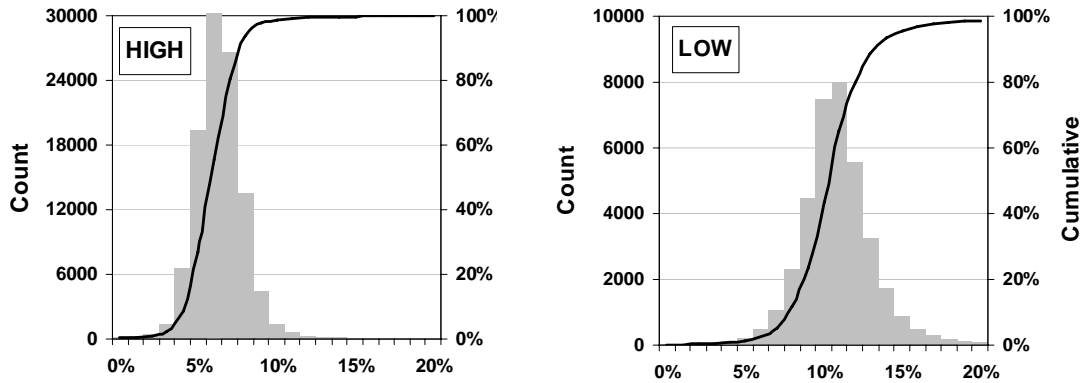


Figure 5: Frequency distribution of the relative offset of sonar versus mag meter.

series of reverse flotation cells. As in this specific example neither meter was “nulled” relative to the other, either the high density histogram or the low density histogram may be used as a reference for the other. It follows that the difference between the two meters, sonar and an electromagnetic, has shifted by about 5% with the change in the magnetite concentration. This is believed to be due to the fact that the calibration of the electromagnetic flowmeter is dependent on the actual concentration of magnetite in the flow. A flow meter that is not affected by the detailed make up of the fluids flowing and any particulates therein is a very clear advantage. It is to be noted that in other comparable cases, where non magnetic particulate material is suspended in the flow, the difference between sonar and an electromagnetic flow meter is not affected by the density.

4 MEASUREMENT OF GAS CONTENT

Using a similar, and in many cases the same, hardware platform it is also possible to measure the velocity of sound of acoustic waves propagating in the process piping. The propagation of acoustic waves occurs at frequencies much above the frequency domain of vortical pressure fluctuations, therefore the same sonar processing can be applied to determine both the acoustical velocity and the vortical velocity at the same time. The relation between speed of sound in twophase mixture and the volumetric phase fraction is well known in case the wavelength of sound is larger than the pipe diameter and hence also significantly larger than any process in homogeneities such as bubbles.

4.1 Velocity of sound in liquid/gas mixtures

The mixing rule¹², known as Wood’s equation¹³, expresses the fact that in a mixture the compressibility of the mixture equals the volumetrically averaged compressibility of the pure components. For the mixture density a similar rule holds: the mixture density equals the volumetric average of the pure component densities. Because the velocity of sound in a fluid equals the square root of the compressibility over the density, the two mixing rules can be written as:

$$\frac{1}{\rho c^2} = \frac{\varphi}{\rho_g c_g^2} + \frac{1-\varphi}{\rho_l c_l^2} \quad (4).$$

$$\rho = \varphi \rho_g + (1-\varphi) \rho_l$$

Here, φ is the volume fraction of gas at line conditions, c is the speed of sound, ρ the density and the subscripts g and l refer to the gas and liquid phase respectively. Figure 6 illustrates the relation between mixture sound speed and gas content for an air water mixture. In most industrial processes, at moderate temperature and pressure line conditions, the compressibility of the gas phase is orders of magnitude larger than the compressibility of the liquid phase. Inversely, the density is dominated by the liquid density. As such, it will be necessary to measure the process pressure (when varying) as both the gas density and the

liquid density are significant when determining the gas volume fraction from the mixture sound speed.

Conversely, neither the velocity of sound in the liquid nor the velocity of sound of the gas is a significant factor for mixtures where one phase is gaseous. In fact, the denominator of the first term in the Wood's equation equals the product of process pressure and polytropic exponent of the gas demonstrating that the gas content at line conditions can be determined using the velocity of sound independent of the gas molecular weight or the gas temperature. Hence, the determination of the gas content using mixture speed of sound is independent of the gas type

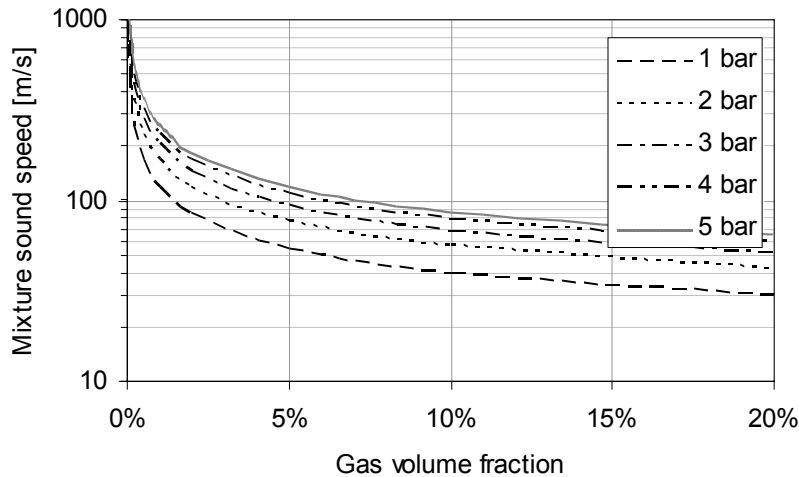


Figure 6: Mixture sound speed vs. gas volume fraction.

and will be accurate irrespective of the type of gas, which can be either air, carbon-dioxide, hydrogen or any mixture thereof.

For mixtures confined to a pipe, a correction to above formulae (4) is necessary in order to account for the relative change of pipe cross sectional area upon internal pressure variation. This so called distensibility¹⁴, D , is an additive term to the first equation of

(4).

The distensibility for thin walled, untethered pipe is given by:

$$D = \frac{2R}{Et} \quad (5),$$

where R is the pipe's internal radius, t is the wall thickness and E the pipe materials' Young modulus. In steel pipe, the distensibility is quite small because of the large value of the Young modulus of steel. In aluminum, glass or even plastic pipe, however, the distensibility must be accounted for. The effect of the pipe distensibility is lower the apparent, measured speed of sound. For instance in a water filled 4" plastic pipe with a wall thickness of 5.5 mm and a modulus of 3.5 GPa, the measured speed of sound will be 420 m/s at zero aeration. This is a striking difference with the free space velocity of sound in water of 1480 m/s.

4.2 Measurement of the velocity of sound

As described above, the relationship between mixture sound speed and entrained air in bubbly liquids is well established. However, as will be developed below, in bubbly flows, these relations are only applicable for the propagation of relatively low frequency, long wavelength sound. While this restriction does not present any significant obstacles for the sonar meter, it does present significant challenges to ultrasonic sound speed measurement devices.

Ultrasonic meters typically operate in 100 kHz to several MHz frequency range. For these meters, entrained air bubbles have length scales on the same order as the acoustic waves generated by the ultrasonic meters. This poses several problems for ultrasonic measuring devices. Firstly, the bubbles scatter the ultrasonic waves, impairing the ability of the ultrasonic meter to perform a sound speed measurement. Secondly, ultrasonic meters rely on information derived from only a small fraction of the cross sectional area of the pipe to be representative of the entire cross section, an assumption that breaks down for flows with non-uniform distributions of in homogeneities, such as stratification of entrained air towards the top of process lines.

Sonar flow meters use an approach developed and commercialized specifically for multiphase flow measurement in the oil and gas industry in which multiphase challenges described above are routinely encountered. Sonar meters measure the propagation velocity of operationally generated sound in the ~100 to 1000 Hz frequency range. In this frequency range, sound propagates as a one-dimensional wave, using the process pipe as a wave-guide. The wavelength of sound in this frequency range (>1 m) is typically several orders of magnitude larger than the length scale of the any bubbles or flow non-uniformities. The long wavelength acoustics propagate through multiphase mixtures unimpeded, providing a robust and representative measure of the volumetrically averaged properties of the flow.

For the sound speed measurement, the sonar flow meter utilizes similar processing algorithms as those employed for the volumetric flow measurement. As with convective disturbances, the temporal and spatial frequency content of sound propagating within the process piping is related through a dispersion relationship.

$$\omega = kc \quad (6).$$

As before, k is the wave number, defined as $k=2\pi/\lambda$ (1/m), ω is the temporal frequency (rad/s), and c is the speed at which sound propagates within the process piping. Unlike disturbances which convect with the flow, however, sound generally propagates in both directions, with and against the mean flow. For these cases, the acoustic power is located along two acoustic ridges, one for the sound traveling with the flow at a speed of $c + V$ and one for the sound traveling against the flow at a speed of $c - V$.

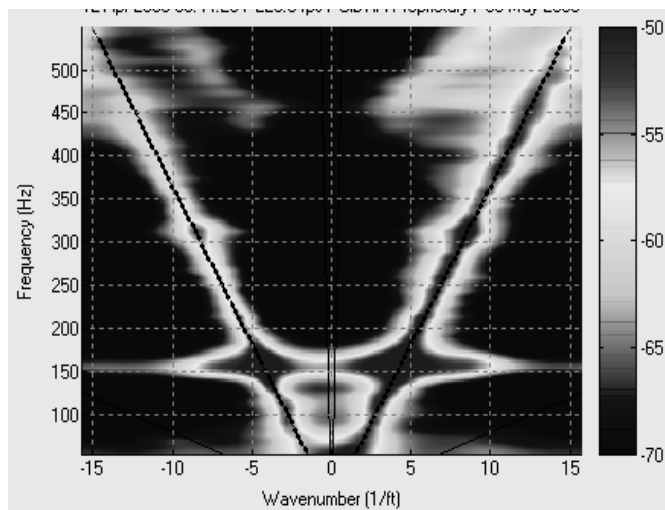


Figure 7: k-w plot for 3% entrained air in water.

Figure 7 shows a k - ω plot generated for acoustic sound field recorded from still water containing ~3% entrained air by volume in an 8 in, schedule 80, vertically oriented Plexiglas pipe. The k - ω plot was constructed using data from an array of strain based sensors clamped to the outside of the pipe. Two acoustic ridges are clearly evident. Based on the slopes of the acoustic ridges, the measured sound speed for this for this mixture was 70 m/s, consistent with that predicted by the Wood equation. Note that adding 3% air by volume reduces the sound speed of the bubbly mixture to less than 10% of the sound speed of liquid only water.

4.3 Industrial applications

As was mentioned in the introduction, there are two applications of the full bore measurement of entrained gas of distinctly different character. First there are those where process aeration must be controlled or where process aeration must be avoided altogether. Examples of such are:

- Natural froth lubricity flows.
- Gas sparging in flotation cells.
- Dissolving carbon dioxide in beverages.
- Entrained air in the thin stock flow to a paper machine's headbox¹⁵.
- Entrained air in filling stations for domestic household products.

Second there are applications where entrained gases negatively affect the process indirectly by affecting other types of meters. Examples of this second kind are:

- Errors in consistency measurement of paper stock.¹⁶
- Errors in the determination of net oil using watercut meters¹⁷.
- Errors in Coriolis determined volume flow as a result of product aeration¹⁸.
- Errors in custody transfer metering resulting from product flashing or aeration.

The sonar based measurement of process aeration using the velocity of sound covers almost 5 orders of magnitude, from 0.01% to above 20%, and is therefore universally applicable to a wide variation of process conditions. By way of example Figure 8 gives the amount of

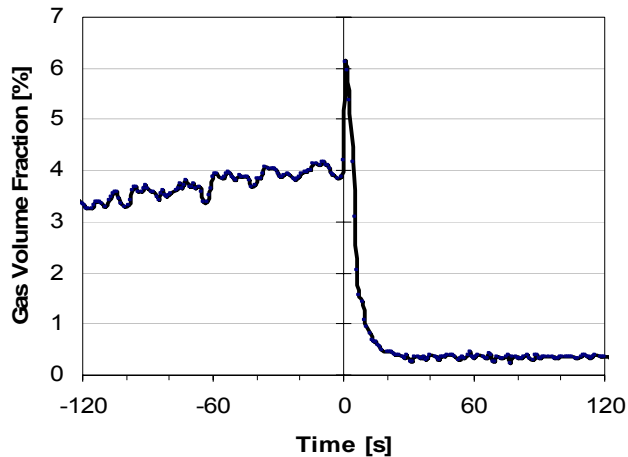


Figure 8: Defoamer dosing application

entrained air in a volume of paper stock just before and just after the dosing of a commercially available defoamer chemical. Given the continuous character of the measurement and given the fact that this full bore measurement is, unlike a sampling system, representative of the true air content the ability to control defoamer dosing presents itself. Such dosing control will not only help reduce defoamer volume it will also help reduce process variability. Thus product quality is positively impacted whilst at the same time cost is reduced.

Conversely, in applications where the amount of air or gas in a process must be kept at a stable level in presence of gas consumption by chemical reaction, this sonar based measurement can provide hitherto unavailable compositional information even on large diameter reactor vessels.. Just reversing the time scale in Figure 8 is sufficient proof of this claim.

5 CONCLUSIONS

Sonar based flow monitoring enables the direct measurement of the convective transport velocity of turbulent eddies in pipe flow and it enables the direct measurement of the velocity of sound propagating through the pipe. Both measurements can be made using an array of strain based sensors clamped to the outside of the process piping.

Evidence is presented to prove that the convective transport velocity of turbulent eddies is proportional the volumetrically averaged mean velocity. A Reynolds number based, empirical calibration is given which yields 0.5% accuracy for Newtonian fluids. Thus, a novel technique based upon naturally occurring phenomena in pipe flow allows the realization of a calibrated flow meter.

Industrial applications of the sonar based flow meter in abrasive and corrosive fluids are discussed. An example in magnetite flows is given where the passive nature of the sonar based instrument proved to be a distinct advantage.

It is shown how the velocity of sound in a twophase medium can be used to derive compositional information such as gas volume fraction. The necessary correction for the effect of the enclosing pipe is developed. The effect of fluid and gas properties on the calculated gas content is discussed. Neither the temperature nor the gas' molecular weight has any influence, only the liquid density and the process pressure must be known.

An industrial application of the gas volume fraction measurement is presented. This defoamer dosing example is one of many which have been found.

6 ACKNOWLEDGEMENTS

The results presented in this work have benefits from several years of collaborative efforts to develop sonar-based flow measurement. The authors gratefully appreciate the efforts of the many colleagues and co-workers that have contributed to results presented herein.

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